Turbulent Drag Reduction in Polymeric Solutions Containing Suspended Fibers

Both polymeric solutions and fiber suspensions have separately been known to exhibit drag reduction under turbulent flow conditions. It has recently been shown that the mechanisms of drag reduction differ appreciably in these two kinds of systems.

The objective of this study was to show that both mechanisms may be exploited concomitantly to achieve unusually low friction factors. Drag reductions in excess of 95% have been obtained in a 2.4-cm tube and there is no evidence that even more dramatic results could not be obtained under optimal conditions.

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SCOPE

The high rates of momentum transport which occur in turbulent fluids dissipate energy at a high rate and may generate objectionable sound levels. Consequently, in turbulent flow processes such as occur in transcontinental pipelines, in flow through high-velocity firehoses, or around submerged objects of naval interest, methods for reducing the drag and the associated sound levels are of interest.

An extensive review of the use of polymeric additives to reduce drag has recently been published by Hoyt (1972a). Use of very high molecular weight additives may result in high levels of drag reduction even at very low polymer concentrations. However, these additives degrade very rapidly due to the high stress levels to which they are subjected in the turbulent field and hence are more useful in one shot applications, such as flow through a firehose, than in flow through lengthy pipelines. The mechanism of drag reduction is believed to reside in the viscoelastic properties of the dilute polymer solutions. These properties may be shown to reduce the radial transport rates in the eddies near the wall of the duct or object (Seyer and Metzner, 1969) as well as the drag on the wall during the locally-unsteady flow conditions characteristic of turbulent processes (Ruckenstein, 1971; Hansen, 1971, 1972, 1973). The fact that these drag reduction mechanisms are wall-region processes brings with it the important disadvantage that the percentage reduction in drag obtainable decreases with increasing scale of the system unless sufficient polymer is used to obtain saturation or asymptotically limiting levels of drag reduction. Alternative (and appreciably different) formulations of these asymptotic or limiting drag reduction levels have been provided in the papers of Virk et al. (1967 and 1970) and Seyer and Metzner (1969).

Although the discovery that particulate suspensions reduce turbulent drag to levels below those experienced with the carrier fluid alone predates the study of this phenomenon in polymeric solutions by 4 decades (see, for example, Vaseleski, 1973; Mysels, 1971) a fundamental mechanistic understanding of this process, in the important region of high Reynolds numbers, is still largely lacking. However, Vaseleski (1973) has recently been able to show, using fiber suspensions, that in ducted flows important changes in turbulent momentum transport occur in the turbulent core.

Since the mechanisms of turbulent drag reduction are so different in suspensions and in polymeric solutions, it is attractive to inquire whether or not both mechanisms, exploited concomitantly, may not yield greater approaches to reversibility (zero drag) than either used alone.

CONCLUSIONS AND SIGNIFICANCE

1. The results obtained at a Reynolds number of 10^5 using one particular system (see text) are typified by the following data:

Nominal tube diameter	Percentage reduction in drag coefficient, using Fiber additive alone	Polymeric additive alone	Both together
2.5 cm	21	50	89
5.0 cm	18	40	78
7.0 cm	22	27	63

It is seen that both additives used together provide for more than a linearly additive effect; the dependence on the scale of the system, zero with fibrous additives alone, is not as adverse with both additives as with the polymer alone.

2. It appears that the strong influences of polymer molecular weight (hence the great susceptibility to degradation) are reduced in mixed systems.

3. The drag reduction obtainable utilizing the mixed systems may substantially exceed the limiting drag reduction asymptotes developed for polymeric systems alone.

4. In physical processes which are pressure-drop limited, the increase in velocity or flowrate obtainable on use of both additives is approximately 6-fold under the conditions studied to date and may increase very rapidly with further increases in flowrate. Correspondingly, in power-limited systems 3-fold increases in velocity occur at constant levels of power input. These are clearly major effects which suggest the further development of this discovery may well be warranted for the purposes of elucidating mechanisms and developing optimal conditions for exploitation.

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Table 1. Capillary Tube Characteristics

Tube no.	Material	Length L, cm	Inner diam. D, cm	Aspect ratio, L/D
1	Stainless steel	58.0	0.0586	990
2	Stainless steel	29.9	0.1063	281
3	Stainless steel	104.3	0.2537	411.1
4	Stainless steel	105.2	0.1686	623.6
5	Polymethyl methacrylate	179.3	0.4624	387.8
6	Polymethyl methacrylate	186.2	0.6359	292.8
7	Polymethyl methacrylate	186.6	0.9779	190.8
8	Polymethyl methacrylate	458.2	1.2641	362.5

APPARATUS AND TEST MATERIALS

Pressure drops were measured manometrically as a function of flow rate through pipes of rigid polyvinyl chloride having diameters of 2.42, 4.87, and 7.03 cm. The manometer taps provided for entrance lengths of 82, 41, and 28 diameters, respectively, upstream of the first tap. Use of such comparatively short entrance lengths was made possible by noting that the region of primary interest is that of high Reynolds number in which entrance flow effects are constrained to a small number of pipe diameters. These taps were of large diameter (0.63 cm) to eliminate clogging by the fibers used as verified by frequent disassembly and inspection as well as by repeat measurements in random order. Extensive and careful calibrations were used to show these do not appear to lead to erroneous pressure readings. The three test sections were fitted into a recirculating loop containing a storage tank, a Moyno 2L10 bladeless pump, and a Foxboro magnetic flow meter. Cooling coils in the storage tank maintained fluid temperatures constant within 1°C.

Viscometric measurements were performed on all fluids used by means of a capillary tube viscometer. A variety of capillary tubes was used (Table 1); the several largest tubes were necessary for use with the nylon fiber suspensions as these clogged the smaller ducts.

The choice of fibrous additives to be used was guided by the extensive data on fiber effectiveness published by Hoyt (1972b). Two chrysotile asbestos fibers and one set of nylon fibers were chosen. The latter were supplied by Microfibers, Inc., with a diameter of 20μ and an aspect ratio of 100. One asbestos fiber was from Turner Brothers Asbestos Company and the other from Johns-Manville Company (labelled 3T12). Turner Brothers report their asbestos fibril diameter as 30-40nm and the mean aspect ratio to be 4×10^4; no dimensions are available for the Johns-Manville asbestos.

For both asbestos fibers, addition of surfactant was found to be indispensable for well-dispersed solutions. The surfactant used in the two asbestos fiber suspensions was Aerosol OT obtained from Fisher Scientific Company. The concentrations of the surfactant in the solution was 0.8% for Turner Brothers fibers, as suggested by the manufacturer, and 0.25% for the Johns-Manville asbestos as recommended by Peyser (1972).

The polymeric additive used in this study was Separan AP30, a partially-hydrolyzed polyacrylamide manufactured by Dow Chemical Company. In the present study only one concentration (150 wppm) was used.

Since the results may be sensitive to degradation and to dispersion of the solids, the experimental techniques employed must be described exhaustively. For each test solution the storage tank was initially cleaned and filled with a weighed amount of water (about 700 kg.). In the earlier phases of the work, the fibers (and the surfactant for the asbestos fibers) were poured into the storage tank directly, and the suspension was stirred with a low speed mixer. In the latter part of this study, when the polymeric additive was introduced, somewhat different procedures were used. Since both the Turner Brothers asbestos fibers and the polymer solution were shear-degradable, only experimental results using fresh samples were collected. For the systems with polymer and nylon fiber suspensions, weighed amounts of polymer were sifted into the water in the storage tank. The polymer solution was then pumped through the test loop for three hours as a pre-shearing procedure before the fibers were added so as to virtually eliminate the shear degradation problem. After the pre-shearing, nylon fibers were poured into the sheared polymer solution and dispersed moderately with a mixer. Finally, for the JM asbestos fibers, soap solution was prepared in the storage tank before the polymeric powder was sifted in and dissolved. The polymer-soap solution was then pre-sheared through the test loop for eight hours. A well-dispersed asbestos fiber suspension, which was prepared separately in about 45 kg. of soap solution, was then poured into the sheared polymer-soap solution for our tests. In order to save material and time, a new (higher) concentration of suspension was made up by making additions to the previous batch. In the solutions with polymer additive, no further polymer was added in these later stages. Thus, the polymer concentrations in the solutions with higher fiber concentrations were lower than the value (150 wppm) we report, having been reduced by about 10 ppm by each fiber addition.

RESULTS AND DISCUSSION

A detailed presentation will be given of the data obtained employing JM asbestos fibers. The results for the other two fiber suspensions, which are generally similar, will be presented more tersely.

Friction factor-Reynolds number data for water and the several surfactant solutions were obtained to establish the experimental techniques and to verify the pipe diameters. The maximum deviation of any point from the von Karman equation for smooth tubes was 4.5%; standard deviations were not computed but may be estimated as between 1 and 2%.

Rheological data for the polymer plus surfactant solutions, for JM fibers suspended in surfactant, and for fibers plus surfactant and polymer, are all presented in Figures 1 and 2. These data, at the stress level corresponding to each experimental turbulent flow measurement, enabled computation of the generalized Reynolds number appropriate to the nonlinear fluid properties. As a matter of fact, however, only a few of the turbulent stress levels were below 20 dynes/cm^2. Above this stress level one sees that the systems used later with polymer (800 ppm of fiber or less) were nearly Newtonian in their viscometric behavior and the distinction in Reynolds number is only

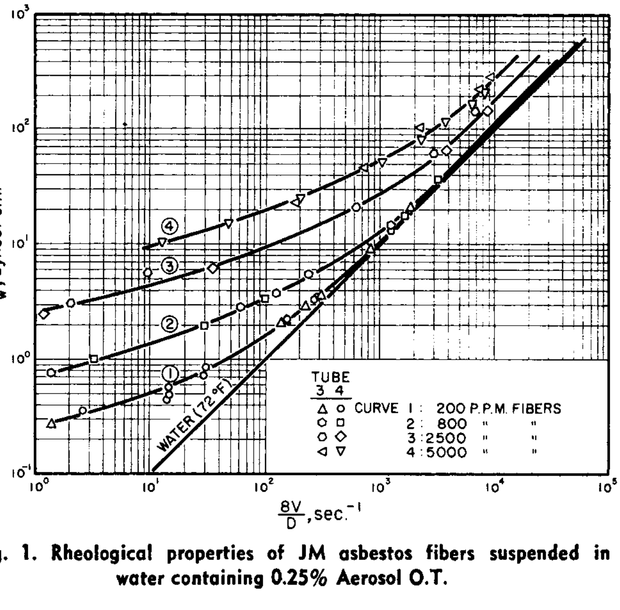

Fig. 1. Rheological properties of JM asbestos fibers suspended in water containing 0.25% Aerosol O.T.

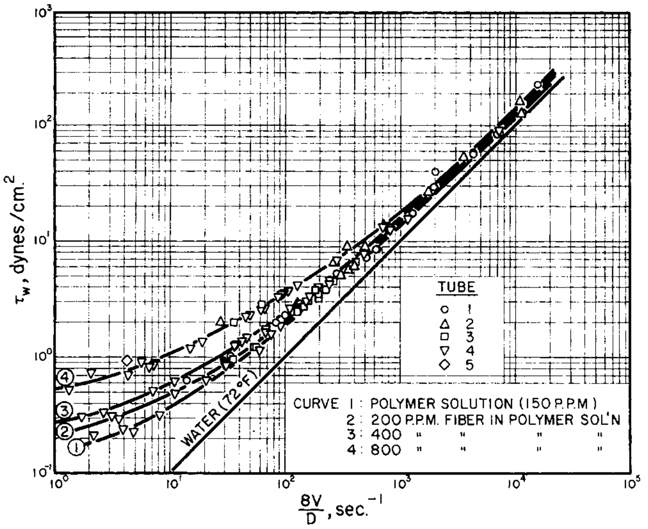

Fig. 2. Rheological properties of polymer solution and its mixtures with JM asbestos suspensions. All systems contain 0.25% Aerosol O.T.

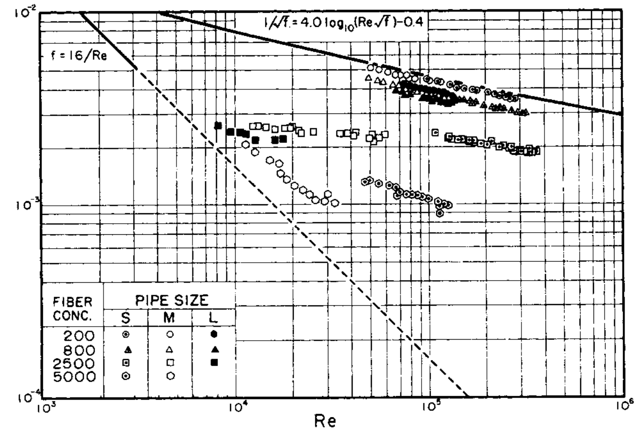

Fig. 3. Friction factor–Reynolds number relationships for the JM asbestos suspensions of Figure 1. S, M and L denote the 2.4-, 4.9-, and 7.0-cm pipe diameters, respectively.

a minor one.

Figure 3 gives friction factor-Reynolds number data for the asbestos suspensions and Figures 4 to 6 show data for the polymer-surfactant solution and for polymer-surfactant-fiber mixtures. These results show surprisingly high levels of drag reduction in the mixed polymer-fiber systems: an 89% reduction in drag coefficient is achieved in the 2.4-cm pipe at a Reynolds number of 10^5 using 800 ppm of fiber in addition to the polymer. Although the reduction in drag is less in the larger pipes it is still significant, as summarized in Table 2.

It is worth noting the roles of the surfactant and of polymer concentration. When surfactant is added to a polymeric solution it may change the rheological properties appreciably, presumably due to the formation of colloidal supramolecular aggregates. Our moderately extensive attempts to exploit this phenomenon for turbulent drag reduction purposes have not, however, been very successful to date. For the present system of polyacrylamide and Aerosol OT removal of the surfactant only results in about a 50% increase in drag coefficient at a Reynolds number of 10^5, and even less at lower flowrates. Thus the surfactant effects, although easily measurable, are not impressive. On the other hand, a 20-fold increase in polymer concentration will decrease the drag coefficient to nearly the same levels as those of the polymer-fiber mixtures of Figures 4 to 6, as shown by Metzner and Park (1964). In this case the viscosity of the system is increased inordinately, however, and the actual reduction in drag, at a given flow rate, is not very great at all. The results of Figures 4 to 6 are unique, therefore, not in themselves but when referred to the small viscosity

Table 2. Reductions in Turbulent Drag Coefficient in Polymer-Fiber Mixtures (JM Fibers)

Nominal pipe diam.	Reynolds number	Fiber conc., wppm*	Percentage reduction in drag coefficient obtained using Fiber additive alone**	Polymeric additive alone**	Both additives together**
2.4 cm	2×10^5	200	2.5	64	73
		800	19	64	92.5
	10^5	200	4.5	50	64
		800	21	50	89
4.9 cm	10^5	800	18	40	78
7.0 cm	10^5	200	13	27	44
		800	22	27	63

* Parts per million by weight.
** All systems contain 0.25% surfactant.

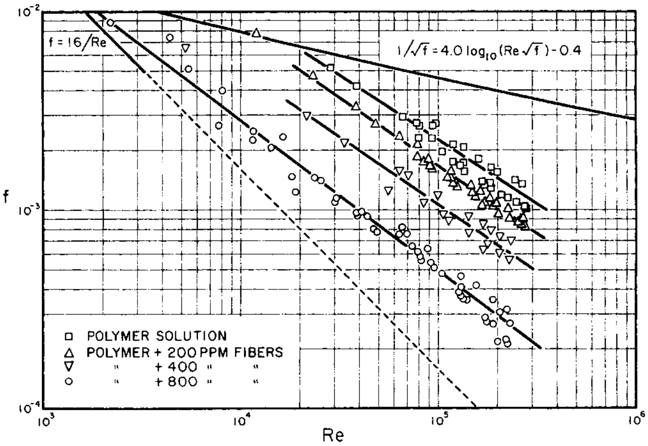

Fig. 4. Friction factor–Reynolds number results: polymer and JM fibers in 2.4-cm pipe. All systems contain 0.25% Aerosol O.T.

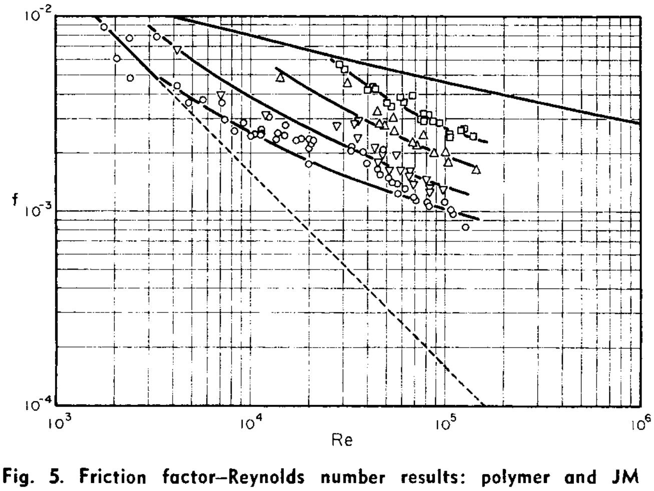

Fig. 5. Friction factor–Reynolds number results: polymer and JM fibers in 4.9-cm pipe. Legend as in Figure 4.

changes noted in Figure 2.

Figure 7 depicts drag coefficient-Reynolds number results for the Turner Brothers asbestos, for the polymer and for the two together. The arrows on the suspension data are meant to indicate the nonrandom order in which these measurements were made since there is a progressive degradation of the particles. Since an undergraded polymer was used in these runs, it alone, at the high concentration used, shows very high levels of drag reduction. Nevertheless, the addition of 200 ppm of fibers decreases the drag by a further appreciable amount (although less than linearly—see Table 3). At the highest Reynolds number reached in the 2.4-cm. tube (2.35×10^5) the reduction in drag coefficient achieved in the mixed system is over 95%.

Figures 8 and 9 and Table 3 summarize the data obtained for nylon suspensions with degraded polymer and no surfactant. Very high nylon fiber concentrations are required for any appreciable drag reduction. Nevertheless, the mixed systems show interesting results: Even though the polymer alone may not reduce drag (in the larger pipes and lower Reynolds) it still possesses the ability to augment the drag reduction of the fiber suspension.

A useful comparison with prior work using polymeric additives may be afforded by Figure 10. This figure confirms a variety of earlier evidence that the maximum drag reduction asymptote of Virk et al. (1970) may require some modest reworking. However, much more important is the fact that the polymer-fiber mixture exhibits far greater drag reductions (larger $1/\sqrt{f}$ values) than given by that asymptotic relationship.

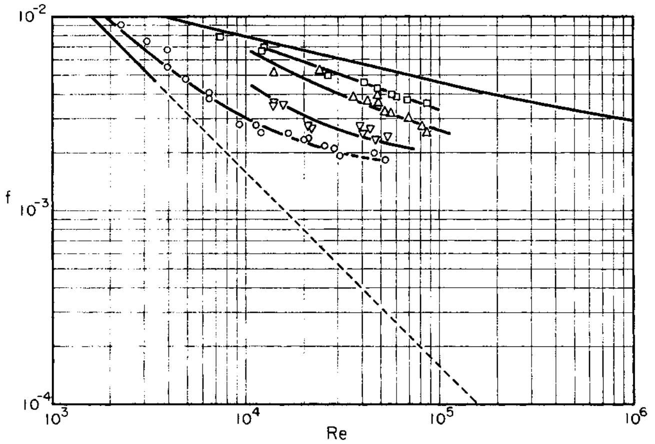

Fig. 6. Friction factor–Reynolds number results: polymer and JM fibers in 7.0-cm pipe. Legend as in Figure 4.

The figures presented thus far, utilizing $f - Re$ coordinates, are most useful from a mechanistic viewpoint, but they also intermingle the measured reduction in drag with the rheological properties of the fluid. To eliminate the latter it has been shown that plots of the actual percentage drag reduction, at a given fluid velocity, may be

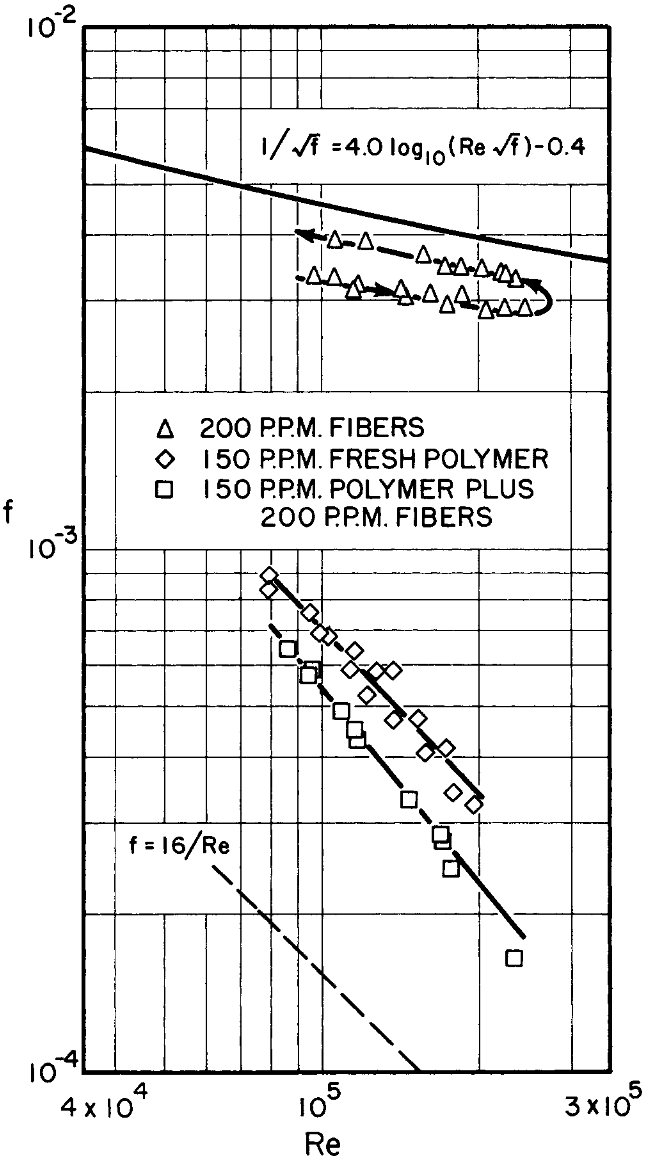

Fig. 7. Friction factor–Reynolds number results: Turner Brothers fibers and polymer in 2.4-cm pipe. All systems contain 0.8% surfactant.

TABLE 3. REDUCTION OF TURBULENT DRAG USING TURNER BROTHERS ASBESTOS OR NYLON ADDITIVES

Nominal pipe diam.	Reynolds number	Fiber conc., wppm	Percentage reduction in drag coefficient using Fiber additive	Polymeric additive alone*	Both additives together*
2.4 cm	2×10^5	200 TB	14-27**	91	94
	10^5	200 TB	13-22**	84	88
4.9 cm	10^5	200 TB	15-28**	67	73
2.4 cm	10^5	1,000 nylon	15	36	63
4.9 cm	5×10^4	1,000 nylon	17	0	36

Polymer concentration: 150 wppm in all cases.
* All asbestos systems contain 0.8% surfactant; the nylon systems contain none.
** Level depends on extent of fiber degradation.

correlated with the friction velocity u^* (Savins, 1967; Whitsitt et al., 1969). Such a plot is given in Figure 11. This figure reveals that:

1. Actual reductions in drag at a given flow rate (not merely in drag coefficient at a given Reynolds number) as great as 94% were obtained and a finite upward trend with increasing friction velocity is still evident at the highest flow rates reached. In the case of Turner Brothers asbestos, plus polymer, a drag reduction level of over 95% was reached and found to be increasing very much more rapidly with increasing friction velocity than the trend shown by the data of Figure 11. The importance of drag reduction levels of this magnitude may be emphasized by noting that at a 95% drag reduction level the pressure drop is only 1/20 of that in the absence of additive. It also represents a 3-fold improvement beyond the maximum drag reduction curve for polymers alone (Figure 13 of Hoyt, 1972a).
2. The addition of fibers to degraded polymer more than offsets the deleterious effects of polymer degradation, suggesting that highly effective composite systems might be developed which are resistant to degradation.

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NOTATION

D = internal diameter of pipe or tube
f = Fanning friction factor (drag coefficient) defined as $f = \tau_w/(\rho V^2/2)$
K' = consistency index for nonlinear fluids, defined by the relation $\tau_w = K'(8V/D)^{n'}$

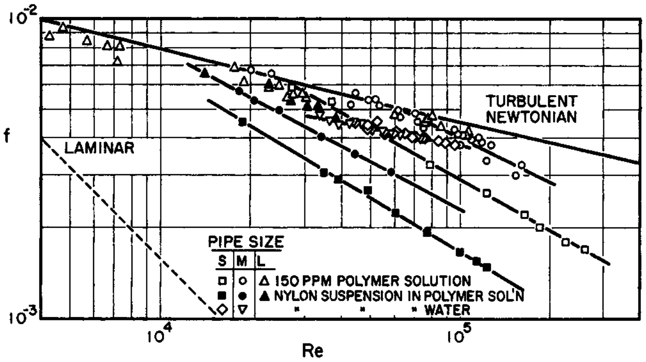

Fig. 8. Friction factor–Reynolds number results for 1,000 ppm nylon fiber suspensions and polymer. Pipe diameters as in Figure 3.

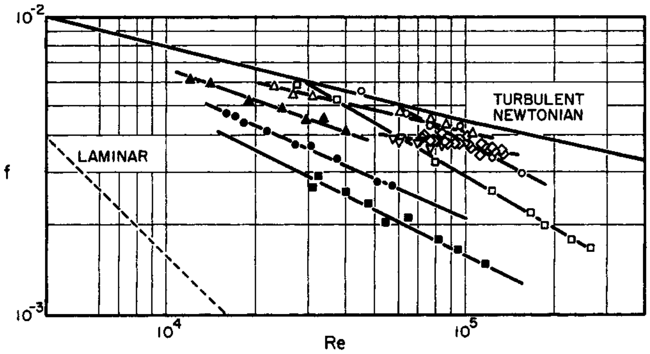

Fig. 9. Friction factor results for 1% (10,000 wppm) suspensions of nylon fibers. Legend as in Figure 8.

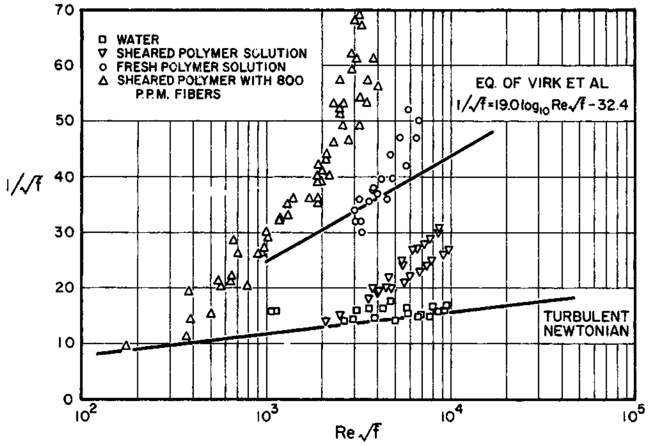

Fig. 10. Von Karman plot of drag coefficient results (all data for 2.4-cm pipe). Surfactant present in all fluids.

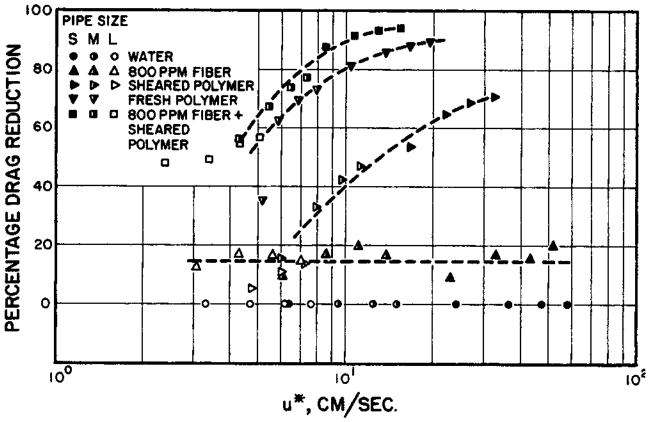

Fig. 11. Drag reduction shown as function of friction velocity. All fluids contain 0.25% surfactant; fiber is JM asbestos. Pipe diameters as in Figure 3.

L = length of tube
n' = flow behavior or nonlinearity index of fluids used
Δp = pressure drop
Re = Reynolds number. For Newtonian fluids $Re = DV\rho/\mu$. For nonlinear fluids $Re = D^{n'}V^{2-n'}\rho/(K' 8^{n'-1})$
u^* = friction velocity $(\sqrt{\tau_w/\rho})$
V = mean velocity
μ = viscosity of Newtonian fluid
ρ = fluid density
τ_w = shearing stress evaluated at wall of tube or duct $\tau_w = D\Delta p/4L$

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A Simple Algorithm for the Time-Optimal Control of Chemical Processes

A simple algorithm for the time-optimal control of chemical processes during setpoint changes, in processes which can be described by a second-order lag plus dead time model, is described. Knowledge of the unsteady state model parameters is not required because the algorithm uses a dimensionless phase plane on which the switching curves are independent of system parameters for a given forcing function. The algorithm gives the parameters of a second-order lag plus dead time model as a byproduct of the setpoint change. It is easily tuned and is relatively insensitive to changes in the process dynamics. The algorithm does not require a large computer or long computing times and has been implemented on both analog and digital computers in controlling computer simulated systems.

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SCOPE

The time-optimal control problem is basically one of determining the control action that will drive a process from an initial state to a specified final state in minimum time. A time-optimal setpoint change for a second-order system is accomplished by applying full forward forcing at time $t = 0$, switching to full reverse forcing at time t_1, and returning to conventional setpoint control at t_2. The problem is to determine the times at which to make the switches, t_1 and t_2.

Time-optimal control offers economic advantages for many types of chemical processes. At plants operating under supervisory computer control, the computer often makes setpoint changes. When the supervisory computer specifies more profitable operating conditions, it is desirable to bring the plant to those new conditions as quickly as possible. This is particularly important if the time between the plant disturbances which cause a change in the optimum operating conditions is comparable to the time required to line the plant out at a new set of operating conditions. In the operation of batch processes, step changes in process variables are often required. Undue delay in completing these changes decreases the capacity of the unit and should be avoided.

In attempting to apply time-optimal control to a process, two obstacles are encountered: (1) An unsteady state mathematical model of the system is required and (2) a nonlinear, multipoint, boundary-value problem, with unspecified final time must usually be solved in order to determine the switching times.

Most of the existing methods of obtaining the time-optimal switching times avoid the boundary-value problem but require prior knowledge of unsteady state model parameters. The objective of this study was to develop an algorithm that does not require the knowledge of these parameters.

CONCLUSIONS AND SIGNIFICANCE

The results obtained show that it is possible to generalize the solution of time-optimal control problem into an algorithm that does not require knowledge of unsteady state model parameters. In the traditional phase plane representation of the time-optimal control problem, the switching curves that determine t_1 are functions of the

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